Gov 51: Bayes Rule

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QAnon

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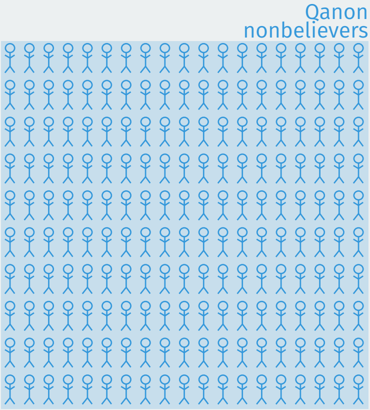
- Common response: probably believes in QAnon since believers tend to be Republicans.
- **Base rate fallacy**: ignores how uncommon QAnon believers are!

Visualizing QAnon support



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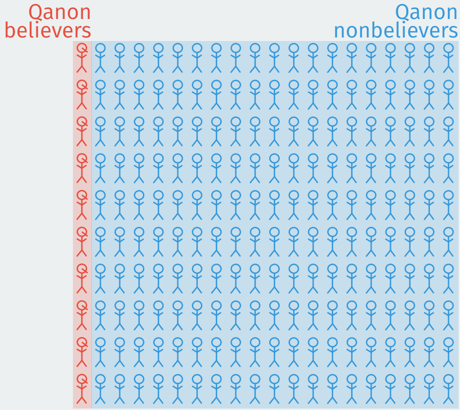

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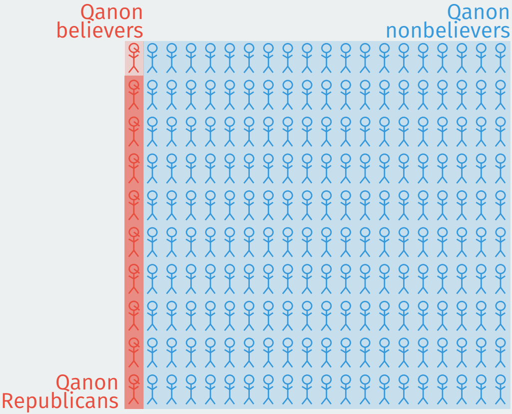

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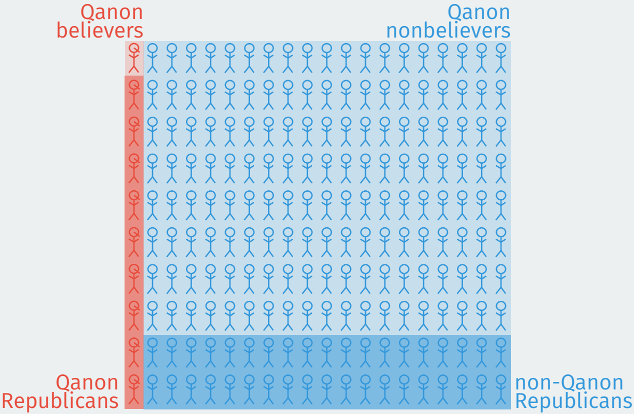

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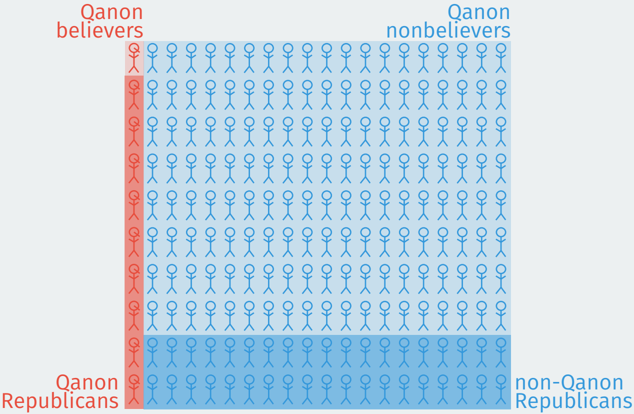

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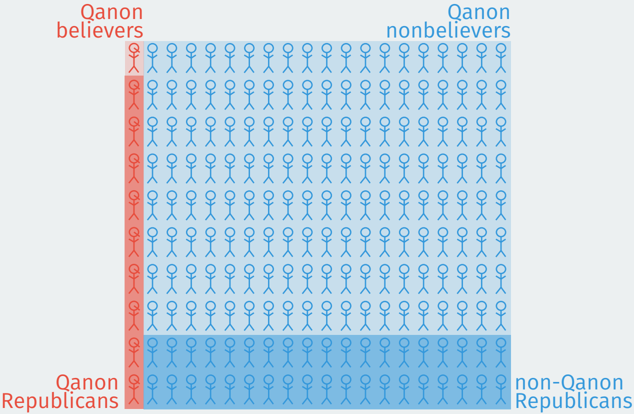

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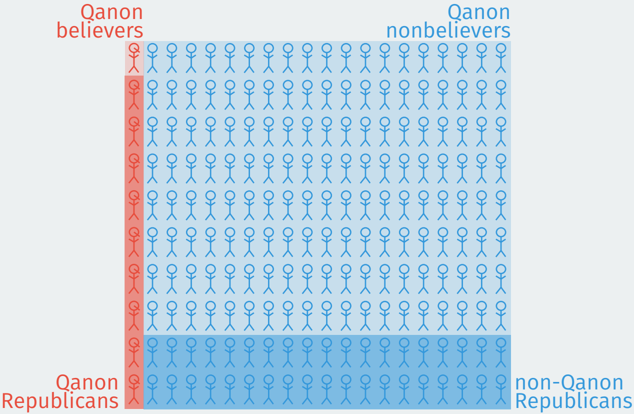

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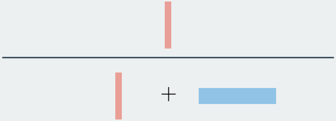

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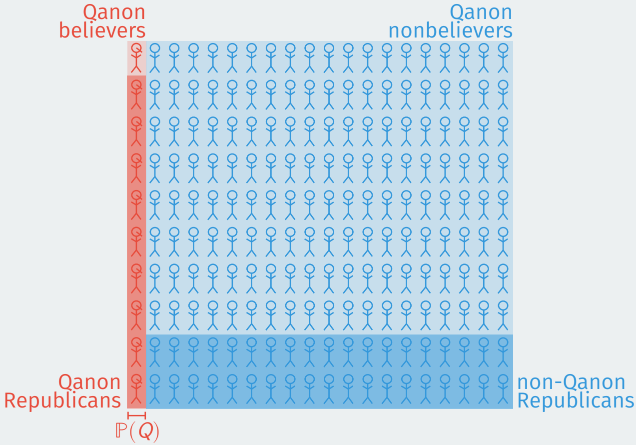
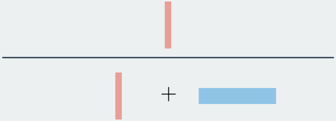


Chance a random Republican believes QAnon = 

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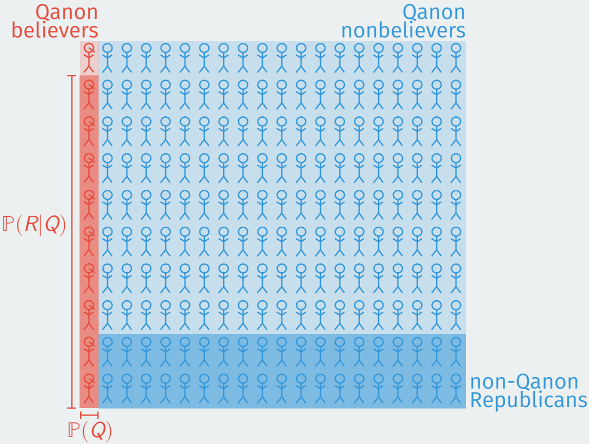

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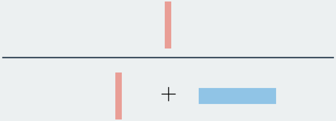

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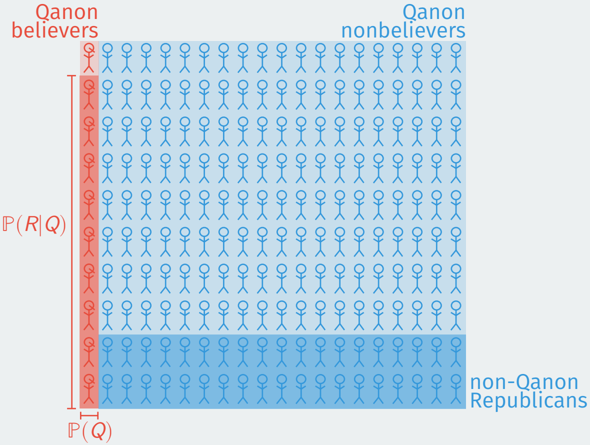

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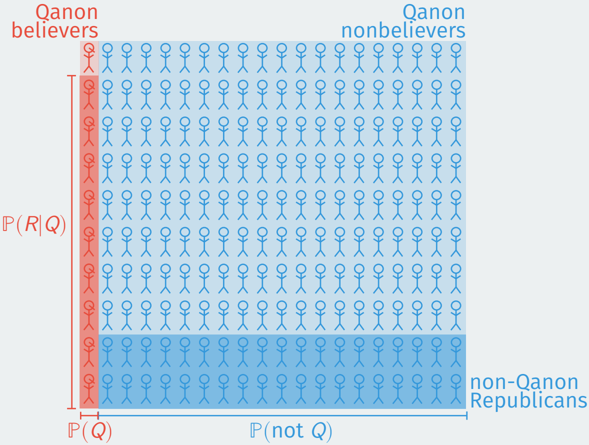

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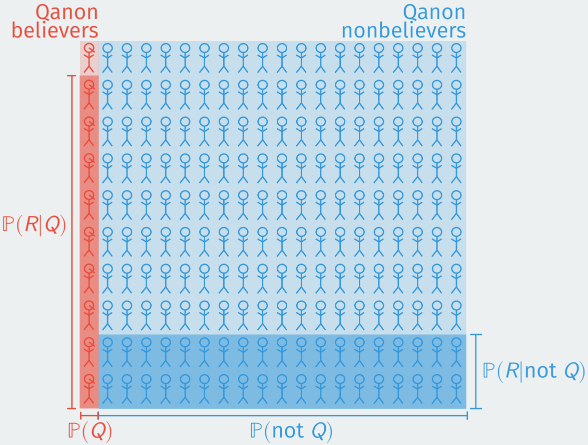

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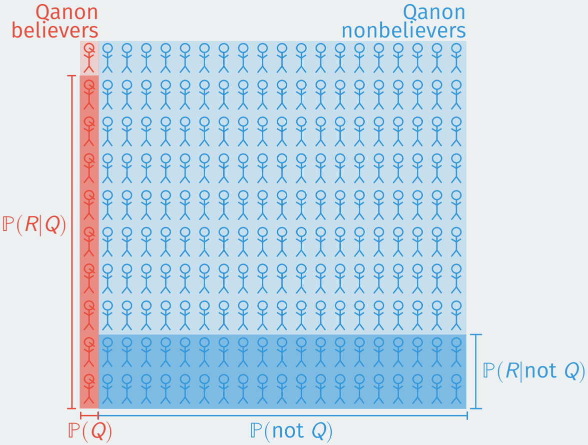

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  - How does the evidence change the chance of the hypothesis being true?

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    - $\mathbb{P}(C) = 0.001$ rough prevalance of active COVID cases.

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$$\mathbb{P}(C \mid PT) = \frac{\mathbb{P}(PT \mid C)\mathbb{P}(C)}{\mathbb{P}(PT)} = \frac{0.7 \times 0.001}{0.051} \approx 0.014$$