Gov 51: What Is A Hypothesis Test?

Matthew Blackwell

Harvard University

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 - Present cups to friend in a **random** order
 - Ask friend to pick which 4 of the 8 were milk-first.

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- \rightsquigarrow the guessing hypothesis might be implausible.

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- Probabilistic proof by contradiction: try to "disprove" the null.

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- Data: poll has $\overline{X} = 0.44$ with n = 1363.

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Simulations of the null distribution



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mean(trump_shares < 0.44) + mean(trump_shares > 0.51)

[1] 0.01

Two-sided p-value



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 - *p* < 0.01 "highly significant"

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 - · Missed out on an awesome finding